Finite Element Analysis of Large Spur Gear Tooth and Rim With and Without Web Effects-Part II

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Abstract

In part I of this paper a program was developed using the ANSYS Parametric Design Language (APDL) to generate 1, 3, and 5 tooth segment finite element models of a large low addendum spur gear. The finite element models are constrained on radial sides in the rim portion and the inside rim surface is left unconstrained. An appropriate load is applied at the highest point of single tooth contact (HPSTC) for the 2-D finite element models and the entire load is applied on a line of nodes located along the face width in the 3-D finite element models. The two and three dimensional models of 1, 3 and 5 tooth segments are analyzed by varying $\beta$, which is the ratio of rim thickness to total tooth height. The effect of web on bending and equivalent stresses is also studied in the 3-D models. The maximum equivalent stresses and maximum bending stresses obtained for different values of $\beta$ in two dimensional and three dimensional models are compared. Three dimensional models with and without constraints at rim-web interface are also compared in 1, 3 and 5 tooth segments. Based on these results and comparisons an appropriate rim thickness is suggested and importance of web is discussed.

Introduction

This paper presents the results of the analysis of 1, 3 and 5 tooth segments. Bending stresses on the tooth profile and rim bottom surface are studied on the tooth subjected to loading for 3 values of $\beta$. The effect of rim thickness on maximum value of bending stress and maximum von Mises stress is studied for 12 different values of $\beta$ in 1, 3 and 5 tooth segments. The 3D models are also studied for the effect of web. A web of 2 inch thickness was considered and to include that affect, the nodes at the rim and web interface at 1 inch distance on either side from the middle of the face width were completely constrained. The bending stresses in these
models were also compared with the 3D models without constraints at the rim and web interface. In the 3D models the location of maximum stress also varies along the face width. In this paper, comparisons for maximum von Mises stress and maximum bending stress was made for the following cases.

1. Comparison of tooth profile and rim bottom surface stresses in the middle tooth of 1, 3 and 5 tooth segments for rim thickness equal to tooth height.
2. Comparison of maximum von Mises stress and maximum bending stress in 1, 3 and 5 tooth segments for 12 different values of $\beta$ in 2D and 3D models.

The comparison plots and the results of the comparison are presented and an appropriate value of rim thickness is suggested. For simplicity results of the case with rim thickness equals tooth height are presented. The results of the cases with lowest and highest values of rim thickness and three dimensional analyses were also studied in this work. The gear on which the analysis is performed is not a specialized gear, so precise comparison was not possible but the results followed the same trend as that of the AGMA 2001-C95 charts. The terminology used to describe the position on profile and rim of the middle tooth is shown in the following Figure 1.

![Figure 1 Terminology in the Gear Tooth](image)

**Comparison of Tooth Profile Stresses**

The Figure 2 shows the tooth profile stresses of the loaded tooth in one, three and five tooth segments. T1 and T2 represent the corresponding points on the tooth profile shown in the Figure 1. In one tooth segment stress at T1 i.e., at the root, is tensile due to constraints. In three and five tooth segments stress at T1 is highly compressive due to bending of the rim and compression from the left tooth. At the nodes that are close to the addendum circle arc bending stress becomes minimum. When approaching the node at which load is applied i.e., HPSTC there is a sudden drop in the stress value. As one moves towards T2 on the profile, the stress at the fillet close to T2 is lower in magnitude when compared to the compressive stress at the fillet close to T1.
Comparison of Rim Bottom Surface Stresses

Bending stresses at the nodes along the circumference of the rim bottom edge of the tooth subjected to loading in one, three and five tooth segment are plotted in the Figure 3. In one tooth segment the stresses at R1 is compressive due to the constraints. While in three and five tooth segment models it is tensile due to the flexibility of the rim. The peak tensile stress on the rim bottom edge occurs at the nodes located radially below the compressive side fillet. For lower values of rim thickness the tensile stress in this region are as significant as the bending tensile stress at the fillet.
Comparison of Maximum von Mises Stress

The influence of rim thickness on maximum von Mises stress is summarized in Figures 4, 5 and 6 in 2D and 3D models of one, three and five tooth segments. The von Mises stress was smaller in the case of 3D than the plane stress case. A stress difference of less than 10% was observed due to the stiffness in the axial direction, which could be accounted for lower stresses in the 3D model.
Figure 4  Comparison of 1, 3 and 5 Teeth Segments, 2D Models

Figure 5  Comparison of 1, 3 and 5 Teeth Segments, 3D Models without Web Constraints
The first column of the Table 1 represents the ratio of rim thickness to tooth height value. The rest of the columns show the values of maximum von Mises stress in 1, 3 and 5 tooth segment two and three dimensional models. It can be seen that von Mises stress was smaller in the case of 3D than the plane stress case. A stress difference of less than 10% was
observed due to the stiffness in the axial direction, which could be accounted for lower stresses in the 3D model.

**Comparison of Maximum Bending Stresses**

The influence of rim thickness on maximum bending tensile and compressive stress is summarized in the Figures 7, 8, 9, 10 and 11 in 2D models and 3D models with and without constraints in the web of one, three and five tooth segments.

![Graph](image_url)

**Figure 7** Comparison of 1, 3 and 5 Tooth Segments, 2D Models
Figure 8  Comparison of 1, 3 and 5 Tooth Segments, 3D Models Without Web Constraints

Figure 9  Comparison of 1, 3 and 5 Tooth Segments, 3D Models With Web Constraints
Figure 10  Comparison of 3D Models with and without Web Constraints in 3 Teeth Segment

Figure 11  Comparison of 3D Models with and without Web Constraints in Five Teeth Segment.
A comparison has been made to study the effect of rim thickness on maximum bending stresses in 1, 3 and 5 tooth segments for the models without web and with web. It can be seen from figures 10 and 11 that including the web for lower values of rim thickness in designing the gears could be more beneficial. Table 2 presents the transition $\beta$ value of each case at which the gear maximum bending stresses does not vary significantly.

<table>
<thead>
<tr>
<th>Number of Teeth in a Segment</th>
<th>Without Web Constraints</th>
<th>With Web Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tensile</td>
<td>Compressive</td>
</tr>
<tr>
<td>One Tooth Segment</td>
<td>0.65</td>
<td>0.7</td>
</tr>
<tr>
<td>Three Teeth Segment</td>
<td>0.75</td>
<td>1.4</td>
</tr>
<tr>
<td>Five Teeth Segment</td>
<td>0.8</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Summary**

The tooth profile and rim bottom surface bending stresses in the 2D and 3D models for all values of rim thickness were almost equal. In 2D and 3D models the bending stresses at the nodes located radially below the compression side tooth fillet in the rim bottom edge or surface are tensile. For lower values of rim thickness in five tooth segment models, these stresses are as significant as the maximum bending tensile stress at the fillet. In the 3D models the location of maximum stress also varies along the face width. One tooth segment did not show significant effect in the tooth bending stress values. In three and five tooth segments presence of web for lower values of rim thickness reduces the maximum bending stress significantly. Results showed that in the one tooth segment there was not much change in the location of maximum bending stress and there was not much change in the maximum bending stress value with the change in rim thickness. Five tooth segment showed higher bending stress than the three tooth segment due to more flexibility of the rim. In five tooth segment for lowest value of rim thickness the location of maximum bending tensile stress was at the root on left end constraints and as the rim thickness increased it was on the tensile side of the loaded tooth. In all the cases the location of maximum bending stress was located closer to the root for lower values of rim thickness and as the rim thickness increased, this location moves up along the fillet.

**References**

RAVICHANDRA PATCHIGOLLA

Mr. Ravichandra Patchigolla completed his Masters in Mechanical Engineering from University of Texas at San Antonio. He has also a Bachelors degree in Mechanical Engineering from JNT University in India. He has worked as a teaching assistant in the areas of machine element design and finite elements area. He also worked as a lab assistant for the ANSYS and SolidWorks software programs.

YESH P. SINGH

Dr. Singh currently serves as Professor of Mechanical Engineering and Biomechanics Department at the University of Texas at San Antonio. He has served as Chair of Mechanical Engineering (9/1993-12/1996), Chair of ME Graduate study Committee and ME Graduate advisor of Records (9/1998-8/2001), and Director of Engineering Machine Shop (1/1998-3/2002). His teaching and research interests are in the area of Mechanical Design. Professor Singh is a registered professional engineer in the states of Texas and Wisconsin, and is an ASME Fellow.