NONLINEAR DIGITAL CONTROL OF CONTINUOUS UNCERTAIN SYSTEMS

Jeff Langston  Chunjiang Qian  Michael Frye
Department of Electrical and Computer Engineering
University of Texas at San Antonio, TX
email: axu913@my.utsa.edu

Abstract

This paper examines the problem of nonlinear digital control of a continuous nonlinear system. Due to the prevalence of digital controllers, the problem of controlling continuous systems through the use of digital controls is an important application area. This paper examines the use of two types of backstepping controller; one using a cancellation design to cancel out the nonlinearities and one using a feedback domination design to dominate, rather that cancel, the undesired nonlinearities. The feedback domination design is found to be faster in convergence. The nonlinear controller allows for negating the effects of nonlinear terms not previously accounted for due to the use of a linear digital control of continuous systems. The significance of this result is that little information about the nonlinear system is required to stabilize this system.

Introduction

The issue of stabilizing nonlinear systems using digital controllers is common in many industrial applications. The application of digital controllers is important due to many factors including cost, the flexibility of configuration, scalability, adaptability, and the static operation of the controller. Unfortunately, this problem is exceptionally challenging because digital implementation of a controller can cause an unstable output if sampling produces aliasing. Stability of a system with a digitally implemented controller is also affected by large amounts of sampled data and can lead to unstable results. The continuous system must be assumed to be asymptotically stable to achieve a stable result when implementing a discrete controller [1].

This paper is aimed at showing the performance of two distinct controller design methods: backstepping with cancellation and backstepping with domination. The cancellation and domination methods approach the design of the controller in different ways. The backstepping with cancellation method is aimed at canceling any undesirable nonlinearities in the system by introducing a virtual controller to wipe out nonlinear terms and ultimately lead to our system controller. The backstepping with domination method approaches the design of our controller by using the structure of the system to use and exploit certain nonlinear terms. This is done by generating a virtual controller that will couple with the nonlinear terms to make our Lyapunov function negative definite and also contain high order nonlinear terms. The nature of these high order nonlinear terms will drive the system to equilibrium much faster than the controller designed by the cancellation method. The controller designed with cancellation can be
efficient and effective, but this design method will prove to take much longer to stabilize the system when compared to the domination controller.

This paper is organized as follows, in the section labeled Problem Statement, a continuous time nonlinear system will be presented, as well as some main results of designing a controller using backstepping with cancellation and domination in order to stabilize the given nonlinear system. In the simulation section, a discrete controller designed earlier in the paper will be analyzed and compared. Simulating provides results in order to confirm stability. Finally, the conclusion section summarizes the results.

**Literature Survey**

Various control methods have been proposed in the literature to solve the control problem for PVTOL aircraft. In [2], an approximate input-to-output linearization method is proposed to achieve the bounded tracking and stabilization problem. The method proposed in [2] is extended in [3], which shows that a flat output control can also be used for tracking control of PVTOL aircraft in the presence of unmodeled dynamics. In [4], with a saturation technique, a backstepping method is proposed for stabilization of PVTOL aircraft. Besides these methods, there are other control methods, such as a nonlinear small gain method by [5], the forwarding method [6], and bounded control method [7]. Recently, based on the stabilizing law proposed in [7] and finite-time control technique [8, 9, 10], we have designed a finite-time observer in [11] which yields an output feedback law in the case when the velocity states are unmeasurable.

**Problem Statement**

This paper examines the following type of nonlinear system

\[
\dot{x}_1(t) = x_2(t) \tag{1}
\]

\[
\dot{x}_2(t) = u(t) + x_1^2(t), \tag{2}
\]

where \( x = (x_1, x_2)^T \in \mathbb{R}^2 \) are the states and \( u(t) \in \mathbb{R} \), and is the control input.

The objective is to design a digital controller for this given system using two methods: backstepping with cancellation and backstepping with domination.

**Cancellation Design**

Based on the objective, a Lyapunov function is picked such that it is positive definite.

\[
V_1 = \frac{1}{2} x_1^2 \tag{3}
\]

Now taking the derivative of \( V_1 \) gives

\[
\dot{V}_1 = x_1 \dot{x}_1 \tag{4}
\]

Now substitute \( \dot{x}_1 = x_2 \)
\[ \dot{V}_1 = x_1 x_2 \] (5)

Introducing a virtual controller \( x_2^* \)

\[ \dot{V}_1 = x_1 (x_2^* + x_2 - x_1^*) \] (6)

Where \( x_2^* = -x_1 \)

Introducing \( \zeta_2 = x_2 - x_2^* \) to simplify notation. Thus, \( \dot{V}_1 = -x_1^2 + x_1 \zeta_2 \). \( \dot{V}_1 \) is not negative definite, which would imply global asymptotic stability. Moving on to finding a positive definite \( V_2 \) and going about the same procedure to attain \( u(t) \). This \( u(t) \) will be designed using cancellation methods, and once input into \( \dot{V}_2 \) will produce a negative definite \( \dot{V}_2 \) for stability.

\[ V_2 = V_1 + \frac{1}{2} \zeta_2^2 \Rightarrow \dot{V}_2 = \dot{V}_1 + \zeta_2 \dot{\zeta}_2 \mid \dot{\zeta}_2 = \dot{x}_2 - x_2^* \]

\[ \dot{V}_2 = -x_1^2 + x_1 \zeta_2 + \zeta_2 (u + x_2^2 + x_2) \]

\[ \dot{V}_2 = -x_1^2 = \zeta_2 (u + x_2^2 + x_2 + x_1) \]

Using cancellation to design \( u(t) \), a negative definite \( \dot{V}_2 \) can be obtained:

\[ u(t) = -x_1^2 - x_2 - x_1 - \zeta_2 \] (8)

\[ = -x_2^2 - 2x_2 - 2x_1 \] (9)

\[ \therefore \dot{V}_2 = -x_1^2 - \zeta_2^2 \] (10)

**Domination Design**

Next, the design of a backstepping controller using the domination method. Using the same procedure as previously for the cancellation method until \( \dot{V}_2 \). Recalling,

\[ \dot{V}_2 = -x_1^2 = \zeta_2 (u + x_2^2 + x_2 + x_1) \] (11)

Using a domination design which will dominate not cancel the high order nonlinearities to design the controller \( u(t) \) such that \( \dot{V}_2 \) is negative definite. Thus, the controller is as follows:

\[ u(t) = -4x_1^3 - 2x_2^3 - 8x_1^2 x_2 - 6x_2^2 x_1 - 4x_1 - 4x_2 \] (12)

**Simulation**

Digitizing each controller in order to simulate them, it can tested if the controllers stabilize the given system. In the case of some airship applications and possibly power plant controllers, the simulation of our controllers sheds light on their relative performance. In these applications, a very fast response time is not necessary or even desired because of the nature of the process and
The following plots run both controllers on the same system, but are based on different design methods. In Figure 1, a simulation of the cancellation controller. The simulation shows that the system is stabilized by the designed cancellation controller, but the states do not reach steady state until about 650 seconds. Figure 2 shows the simulation of the domination design controller. This controller also stabilizes the given system and does so after about 400 seconds. Based on these results, it can be shown that a relatively significant performance gain has been achieved with the domination design method.

Fig. 1: Time History State & Controller Plot for Cancellation Controller
Fig. 2: Time History State & Controller Plot for Domination Controller

Conclusion

Two controllers have been presented in this paper designed using different methods in order to attain stability of the given system. These controllers were also digitized in order to examine the performance of a digital controller trying to stabilize a nonlinear continuous system. As can be seen in the simulations of these controllers, the domination design method helps us arrive at a controller that stabilizes the system much quicker. The cancellation method cancels the nonlinearities during the backstepping process. The domination process overwhelms these nonlinearities with higher order nonlinear terms in order to drive the system to the origin quickly.

References


